Automated analysis of multiple-pulse particle image velocimetry data

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An application of multiple-pulse particle image velocimetry to the study of particle motion in a two-phase flow through a cylindrical tube bank is described. An algorithm is developed that automatically analyzes the digital, multiply exposed pictures of this flow and determines the particle trajectories, using a linear particle image tracking method. From these trajectories particle velocities as well as points of impact and angles of incidence and rebound for particles that collide with the cylinder surfaces are determined. This algorithm is sufficiently rapid that data can be collected and analyzed contemporaneously.

Key words: Velocimetry, pulsed laser velocimetry, particle image velocimetry.

1. Introduction
An algorithm that automatically analyzes the data from multiple-pulse particle image velocimetry (MP-PIV) is described. In MP-PIV multiply exposed pictures are recorded of particle-laden fluid moving in a pulsing sheet of light. In these pictures the trajectory of each particle appears as a series of particle images. The velocity of a particle along its trajectory can be determined by measuring the displacement of its image between consecutive illumination pulses. In MP-PIV pictures many images are recorded of a small number of particles. This contrasts with the double-pulsed particle image velocimetry technique in which a small number of images, exactly two, are recorded of a large number of particles. These two techniques record different information and are thus useful in different applications: MP-PIV allows the study of the time evolution of particle trajectories, while particle image velocimetry is useful for making instantaneous maps of flow fields.

Several recent applications of the MP-PIV techniques are described in Refs. 2-6. We are currently using this technique to study the fluid mechanics of particle-laden flows through bundles of cylinders and in particular the collisions of the particles with the surfaces of these cylinders. The significant components of our experimental equipment are illustrated in Fig. 1.

The flow system consists of one or more cylinders suspended in a wind tunnel. Typically these cylinders are 1 cm in diameter, and the air velocity at the inlet of the wind tunnel test section is 10 m/s in the horizontal direction. The air is at ambient temperature. The flow is seeded with spherical glass beads, which have been classified aerodynamically into narrow size ranges. Only one size range is used in any given experiment. The smallest bead diameter used is 10 \( \mu \)m, and the largest is 100 \( \mu \)m. The particle loading is very low—fewer than ten particles appear in each multiply exposed picture.

A copper-vapor laser is used to produce the pulsing light sheet, which is visible \(( \lambda = 511, 578 \text{ nm})\). This sheet is 2 mm thick and can be varied in width up to 10 cm. The pulse frequency of the copper-vapor laser can be varied from 1 Hz to 20 kHz to obtain closely spaced but nonoverlapping particle images in the region of the flow that is being studied. For frequencies of \( \leq 5 \text{ kHz} \) the pulse energy is 2 mJ. For frequencies above 5 kHz this energy is inversely proportional to the frequency and decreases to 0.5 mJ at 20 kHz. The pulse duration of the laser is fixed at 30 ns, which is sufficiently short to freeze the motion of 10-\( \mu \)m particles traveling up to 30 m/s.

The pictures are recorded by a high-resolution (1024-line \( \times 1280\)-pixel/line) video camera and digital frame grabber system. The digital frame grabber digitizes the video signal into 256 gray levels and stores the \( 1.31 \times 10^6 \) pixels of the picture in its frame display memory. The frame grabber hardware is resident in a 25-MHz 386-based microcomputer. A photoelectric sensor–digital system was chosen over film as the recording medium for MP-PIV pictures for several reasons, which include higher sensitivity to

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light, convenience, and the possibility of real-time analysis of the data.\textsuperscript{4}

Figure 2 shows a multiply exposed picture typical of those obtained in our study. This picture records the motion of three particles, 90 \( \mu \text{m} \) in diameter, in a vertical plane. The direction of the flow is from right to left. The outlines of two concentric cylinders are visible. The inner cylinder is projected into the flow and is hit by one of the particles. The outer cylinder is part of the supporting structure for the inner cylinder and appears in the background. The laser pulses at a constant rate of 10 kHz. Each particle image spans \( \sim 5 \) pixels, and the distances separating successive particle images are \( \sim 15 \) pixels for the incoming particles and \( \sim 10 \) pixels for the one rebounding from its collision with the cylinder. The length of the picture in the object plane is \( \sim 6 \) cm, and its height is \( \sim 4 \) cm. The horizontal streaks are caused by electromagnetic interference from the copper-vapor laser. For printing purposes the picture shown in Fig. 2 is thresholded and binarized, and its contrast is inverted. The threshold value used is 120, which is the same value used to analyze the picture. The top half of the picture shown in Fig. 2 is cropped since it contains no particle images.

An algorithm that automatically analyzes the motions of the particle in pictures such as in Fig. 2 is implemented in the C programming language on the microcomputer that hosts the digital frame grabber system. This algorithm is described in Section II.

II. Algorithm Description

The overall organization of the analysis algorithm is illustrated in Fig. 3, the algorithm's structure chart.

![Algorithm Structure Chart](image)

In this chart the modules or tasks of the algorithm are arranged from top to bottom and left to right in the order of their execution. The module names used in the discussion below refer to Fig. 3.

A. Configure

The algorithm begins by calling the module CONFIGURE, which enters the user-specified parameters that govern the analysis. These parameters can be divided into three groups. The first is related to particle image detection: \( T \), the particle image gray-level threshold, and \( ND \) and \( XD \), the minimum and maximum particle image diameters. The second is used in trajectory tracking: \( DF \) and \( AF \), the distance and angle factors; \( XS \), the maximum image separation; \( SC \), the precollision slope cutoff; \( PF \), the proximity factor, and \( NL \), the minimum trajectory length. The third group of parameters is used in the analysis of the trajectories: \( HS \) and \( VS \), horizontal and vertical scales; \( LP \), the laser pulse period; and \( X \) and \( Y \), the column and row coordinates of the cylinder center.

The values of the first and second groups of parameters can be determined quickly and easily by using trial and error and the repeated analysis of the first several pictures of a series. These values can also be determined by using the measurement tools provided in image processing software packages. Experience shows that the algorithm is not sensitive to these parameters and that they remain relatively constant as the camera location and the experimental conditions change.

The values of the third group of parameters are directly associated with the experimental setup and must be determined whenever it changes. The horizontal and vertical scales, \( HS \) and \( VS \), are used to convert distances from pixels into laboratory units. These are determined from the picture of a calibration target placed in the object plane of the video camera. Two scaling parameters are required because the pixels of the digitized image are not square.

Particle image displacements are converted into particle velocities by using the laser pulse period \( LP \), which is measured on an oscilloscope. The cylinder center
coordinates, X and Y, are in units of pixels and are determined from the MPPIV picture. If the center of the cylinder is outside the picture, X and Y are determined from the positions of the translation stages on which the video camera is mounted. These stages are controlled by the image acquisition and analysis computer.

B. Find Centroids
The second module executed by the algorithm is FIND CENTROIDS. This module's task is to locate the centroids of the particle images in the MPPIV picture. FIND CENTROIDS repeatedly calls two modules to help. These are SCAN and FILL. SCAN searches one horizontal line of the picture for pixels whose gray-scale indices are greater than or equal to a user-specified threshold T. Each time such a pixel is found a potential particle image is located, and FILL is called to determine its area and centroid.

The filling of the potential particle image begins with the pixel located by SCAN. All pixels whose gray-scale indices are greater than or equal to the specified threshold value and that are contiguous with this seed are filled by setting their gray-scale indices to less than T. Two pixels are considered contiguous if their row index or their column index, but not both, differs by unity. It is necessary to modify the indices of the above-threshold pixels to prevent the detection of the potential particle image on more than one line of the picture. The area of the particle image is the count of the filled pixels, and the coordinates of the centroid are the average row and column coordinates of these pixels.

The row and column coordinates of pixels are integer values. However, floating-point arithmetic is used to calculate the particle image centroids so that they are located with subpixel precision. The uncertainties in the coordinates of any pixel are ±1/2 row and ±1/2 column. These are generally the uncertainties in the coordinates of the particle image centroids.

The area of a potential particle image must fall within the minimum and maximum area limits before its centroid is added to the centroid list. These limits are computed from the user-specified minimum and maximum particle image diameters, ND and XD, assuming circular images. The minimum area limit is used to eliminate noise, such as electromagnetic interference, which is typically smaller than the particle images, and the maximum area limit is used to eliminate large objects, such as cylinder surfaces.

In its search for particle image pixels SCAN processes the picture from right to left and top to bottom, which corresponds to the general sense of the particle motion. As a result some of the entries in the centroid list are naturally ordered, minimizing the searching necessary to form trajectories. Additional time is saved by not scanning every line of the picture. The user-specified minimum particle image diameter ND is used as the vertical scan increment.

SCAN and FILL are able to locate particle image centroids in the as-digitized, gray-scale pictures. The simple threshold detection method employed in these processes requires a contrast between the particle images and the background and requires that this contrast be maintained across the entire picture. However, the use of more complex particle image detection methods increases the analysis time.

Because SCAN and FILL directly access the frame display memory, they are hardware dependent. They are the only hardware-dependent modules in the analysis algorithm.

C. Form Trajectories
The next module called by the algorithm is FORM TRAJECTORIES. This module arranges the list of particle image centroids into trajectories. The formation process for each trajectory begins by calling the module INITIATE, which forms a nascent trajectory composed of three collinear, equispaced centroids.

INITIATE considers all possible pairs of centroids that have not yet been included in a trajectory and whose separation distances are less than or equal to the user-specified maximum image separation XS. The restriction that the centroids be unused is sufficient to prevent the formation of multiple copies of trajectories. The condition that the separation be less than or equal to XS reduces the interference between trajectories during initiation. For each allowed pair of centroids a search is made for the third centroid necessary to initiate a trajectory. This search is illustrated in Fig. 4.

First, the hypothetical trajectory, shown as a solid line, is extended in a linear fashion. This extension is shown as a dashed line. Then a region of interest is established around this extension by using the user-supplied distance and angle factors. The perpendicular dimension of this region is given by 2dAF, where d is the distance separating the two centroids and AF is the angle factor. The parallel dimension is given by 2dDF, where DF is the distance factor. The values of DF and AF used in our application are typically 0.1–0.3 (corresponding to 5–17° for AF). The centroid list is then searched, and the first centroid found that lies within the region of interest completes the initiated trajectory.

At this point one end of the three-centroid trajectory is arbitrarily called the tail, and the other is called the head. The modules EXTEND TAIL and EXTEND HEAD are then called to lengthen the trajectory by adding more particle image centroids to its tail and to its head.

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Fig. 4. Linear projection used to initiate and extend particle trajectories.

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The trajectory extension process is identical to that of initiation, with the exception that if no centroid is found during the centroid list search, the dimensions of the region of interest are increased by 50% and the centroid list is searched again. The growth of a trajectory end terminates when this backup search fails.

Within the algorithm, the linear extension of the trajectories and the establishment of regions of interest are performed by the module LINEAR PROJECTION. The centroid list searches are handled by the module SEARCH CENTROID LIST.

As centroids are incorporated into trajectories their entries in the centroid list are marked. Centroids are allowed to be members of more than one trajectory to handle correctly the intersecting trajectories that have overlapping particle images at their points of intersection.

The number density of the particles in the flow is limited by the trajectory-forming procedure. In general this limit is established by the requirement that the distances between trajectories must be greater than the distances separating the successive images of the particles. The average number of particles entering the field of view of the video camera during each picture is \( N = n_v A \Delta t \), where \( n \) is the number density of the particles, \( v \) is the magnitude of the free-stream air velocity, \( A \) is the area of the laser sheet normal to the flow, and \( \Delta t \) is the exposure time. A reasonable trajectory separation distance would be 20 pixels, which is based on a typical particle image separation distance of 15 pixels. For flow parallel to the scan lines of a 1020-line video picture, \( N \) is \( \sim 50 \).

Typical values of \( v \) and \( A \) are 10 m/s and 0.8 cm\(^2\), and at the U.S. standard video frequency, \( \Delta t \) is 1/30 s. Using these values gives an upper limit on the particle loading \( n \) of 2 lines/cm\(^2\).

However, the limit determined above decreases as the trajectories become increasingly curved and as their frequency of crossing increases. In our application the number of particles in each picture was limited to \( \sim 10 \). This corresponds to an upper limit on the particle loading of 0.4 lines/cm\(^2\).

D. Join Trajectories

After all the trajectories are formed the precollision and postcollision trajectory fragments belonging to the particles that collided with a cylinder must be joined. The first task in this process is to establish the correct sense of time evolution along the trajectories. This is done by calling the modules CLASSIFY and ORIENT PRECOLLISION.

CLASSIFY fits a least-squares line to each trajectory. As Fig. 2 illustrates precollision trajectories have smaller vertical displacements than postcollision ones. Thus, if the slope of the least-squares line for a given trajectory is less than the user-specified precollision slope cutoff SC, the trajectory is classified as precollision.

ORIENT PRECOLLISION identifies the head or beginning of each precollision trajectory as the trajectory end that is closer to the right-hand side of the picture. This orientation is possible because the uncollided particles move from right to left. The postcollision trajectories cannot be oriented because their motion does not have a definite sense.

The actual joining of trajectory fragments is done by the modules POSTCOLLISION HEAD and POSTCOLLISION TAIL, which try to join the head, or if this is not successful, the tail, of each postcollision trajectory to the tail of a precollision one. (The head and tail labels are arbitrarily assigned to the ends of the postcollision trajectories.) Since the particles are not distinguishable, the trajectory fragments to be joined are identified by the procedure that is illustrated in Fig. 5.

First, linear extensions of the tail of a precollision trajectory and one of the ends of a postcollision trajectory are made, and their point of intersection is determined. In Fig. 5 these trajectories are shown by solid lines, and dashed lines are used to indicate their linear extensions. Since the uncollided particles move from right to left, these two trajectory ends can be joined only if this point of intersection lies to the left of the tail of the precollision trajectory.

Then the proximity of the ends of the trajectories is checked. A circle, centered on the tail of the precollision trajectory, is constructed with a radius of \( d'PF \), where \( d' \) is the distance separating the last two centroids of the precollision trajectory and PF is the user-specified proximity factor. Another circle, centered on the end of the postcollision trajectory, is constructed with a radius of \( d'PF \), where \( d' \) is the distance separating the end and next-to-end centroids of the postcollision trajectory. If the point of intersection determined above, the tail of the precollision trajectory, and the end of the postcollision trajectory all lie within the common area of these two circles, the two trajectories are joined. The point of intersection is assumed to be the collision point of the particle with the cylinder.

The two conditions described above were selected because they are sufficient to match trajectory fragments and because they require a minimum amount of input from the user with regard to the cylinder.

As is evident in Fig. 2 particle images close to the cylinder surface are lost in its surrounding back-
ground. As a result the values of PF used in our application were larger than unity, typically in the 5–15 range. It should also be noted that the trajectory fragments are joined on a first-matched first-joined basis. No attempt was made to find the best match; however, this could be easily implemented if warranted.

E. Analyze Trajectories

This module analyzes each trajectory from head to tail, determining the particle’s velocity at each of its images and information related to any particle–cylinder collision.

Velocities along precollision trajectories are determined by using linear backward differences. We started with the second particle image and ended with the last. Linear forward differences are used to find velocities along postcollision trajectories. We started with the first particle image and ended with the next to last. The change from backward to forward differences allows the determination of the velocities closest to the points of impact.

The magnitude of a particle’s velocity is
\[
v = \frac{Md}{\tau},
\]
where \( M \) is the magnification of the imaging system and \( d \) is the displacement of its image centroid during the time \( \tau \) between illumination pulses. The uncertainty in \( v \) is given by \( \Delta v = M\Delta d/\tau \). The uncertainties in \( M \) and \( \tau \) are neglected. Since \( d \) is the difference between two centroids’ coordinates, its worst-case uncertainty is \( \pm 1 \) row or column. For typical values of \( M = 6 \times 10^{-4} \) m/pixel and \( \tau = 1 \times 10^{-4} \) s the maximum predicted uncertainty in \( v \) is 0.6 m/s. The direction of a particle’s velocity with respect to the horizontal is \( \alpha = \arctan(v_v/v_h) \), where \( v_v \) and \( v_h \) are its vertical and horizontal velocity components. For a typical particle velocity of 10 m/s at 45° uncertainties of 0.6 m/s in \( v_v \) and \( v_h \) result in a worst-case uncertainty in the velocity direction of \( \sim 5^\circ \).

The angles of incidence and rebound are determined with respect to the normal to the cylindrical surface that passes through a particle’s point of impact. This normal is found by projecting a line through the user-specified cylinder center coordinates, X and Y, and the point of impact.

III. Results

The MPPIV picture shown in Fig. 2 was analyzed by using the algorithm described above. Figure 6 shows this picture after the analysis was completed. The centroids of possible particle images are marked by solid circles. Solid lines indicate the three trajectories that were formed. The point of impact of the particle that collided with the cylinder is indicated by an open circle, and this point is connected to its corresponding trajectory fragments with dashed lines. At the point of crossing between the top and middle trajectories the overlapping particle images could not be resolved. This produced a small deviation in the middle trajectory.

The velocities determined from the analysis of the trajectories are displayed in a scatterplot, which is Fig. 7. The abscissa of this plot is the picture column index, which is 0 at the left edge of the picture and increases to 1279 at the right edge. The horizontal and vertical components of the particle velocities are distinguished; the individual trajectories are not. From column coordinates 0–600 the velocity components are from the middle particle. The deviation in its trajectory at the crossing point with the top trajectory is evident between 670 and 680. The effect of the collision with the cylinder on the velocity of the top particle is clearly visible between 640 and 740. Above \( \sim 800 \) the velocities of the three trajectories are superimposed.

The particle velocities in the incoming portion of the top trajectory and in the bottom two trajectories are identical and remain constant across the picture. The mean horizontal and vertical components of these velocities are 7.5 and \(-0.5\) m/s. The observed standard deviations of both components are 0.2 m/s. Thus the relative uncertainty of the magnitude of the mean velocity is 3%, and the uncertainty of its orientation is \( 2^\circ \). For the particle in the top trajectory

### Table I. Time Required to Analyze Fig. 2

<table>
<thead>
<tr>
<th>Algorithm Module</th>
<th>Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIND CENTROIDS</td>
<td>3.8</td>
</tr>
<tr>
<td>FORM TRAJECTORIES</td>
<td>2.3</td>
</tr>
<tr>
<td>JOIN TRAJECTORIES</td>
<td>0.1</td>
</tr>
<tr>
<td>ANALYZE TRAJECTORIES</td>
<td>0.8</td>
</tr>
<tr>
<td>Total</td>
<td>7.0</td>
</tr>
</tbody>
</table>

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the angles of incidence and the rebound with the
cylinder are in the object plane $49^\circ$ and $55^\circ$, respec-
tively.

The times required by the algorithm to analyze Fig.
2 are given in Table I.

IV. Concluding Remarks

An application of MPPIV and an algorithm that
automatically analyzes the multiply exposed pictures
it generates have been described. This algorithm
reconstructs the particle trajectories, using a simple
linear tracking method. From these trajectories, par-
ticle velocities, as well as points of impact and angles
of incidence and rebound for particle collisions with
cylindrical flow boundaries, are determined. In the
application described here the algorithm has proved
to be easy to use, reliable, and rapid.

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